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14. ABSTRACT This work is aimed at understanding the effect of pre-stressing the constituents of composites with a view towards enhancing the overall load carrying capacity of the composites and studying the effect of microstructural morphology on the overall mechanical performance of composites through the use of proper constitutive models for each constituent. The ultimate goal is to carry out a novel theoretical and computational study that assesses the overall performance of composites due to pre-stressing, at the microscopic level, one or more of the components of					
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## Report Title

The Effect of Pre-stresses and Microstructural Morphology on the Overall Mechanical Properties of Composites

### ABSTRACT

This work is aimed at understanding the effect of pre-stressing the constituents of composites with a view towards enhancing the overall load carrying capacity of the composites and studying the effect of microstructural morphology on the overall mechanical performance of composites through the use of proper constitutive models for each constituent. The ultimate goal is to carry out a novel theoretical and computational study that assesses the overall performance of composites due to pre-stressing, at the microscopic level, one or more of the components of a composite with a view towards the design and development of such bodies. We conducted two-dimensional and three-dimensional studies on composite systems comprising of a) brittle inclusions (glass, ceramics) in a ductile matrix (metals, polymers) and b) ductile inclusions in a brittle matrix, and we considered both elastic and viscoelastic response of the constituents. The results suggest that by applying compressive prestress on brittle inclusions, we can expect an increase in load carrying capacity of the structure undergoing externally tensile loads. Also, by prestressing the inclusions, the hoop stress difference at the interface changes in a manner that is of great value when debonding is the main concern.

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**Non Peer-Reviewed Conference Proceeding publications (other than abstracts):**

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1	1
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<u>NAME</u>	<u>PERCENT SUPPORTED</u>	Discipline
Huanlin Zhu	1.00	
<b>FTE Equivalent:</b>	<b>1.00</b>	
<b>Total Number:</b>	<b>1</b>	

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### Names of Post Doctorates

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<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
KR Rajagopal	0.40	
Anastasia Muliana	0.80	
<b>FTE Equivalent:</b>	<b>1.20</b>	
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## **Scientific Progress**

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# Final Project Report

ARO Grant

The Effect of Pre-stresses and Microstructural Morphology on the  
Overall Mechanical Properties of Composites

PIs: Anastasia Muliana and KR Rajagopal

Department of Mechanical Engineering,  
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College Station, TX 77843

September 30, 2013



## 1) Forward

The multi-component and multi-scale nature of composites presents heterogeneities at various scales, leading to stress discontinuities and often delamination at the interface between the different constituents. When delamination occurs, it disrupts load transfer between constituents, reducing the overall performance of composites and shortening the life of composites. Strong bonding at the interface by the development of new adhesives and/or surface treatments of the inclusions could lead to fracture or cracking in the fiber and/or matrix constituents, especially when brittle constituents are considered. This work is aimed at understanding the effect of pre-stressing the constituents of composites with a view towards enhancing the overall load carrying capacity of the composites and studying the effect of microstructural morphology on the overall mechanical performance of composites through the use of proper constitutive models for each constituent. The ultimate goal is to carry out a novel theoretical and computational study that assesses the overall performance of composites due to pre-stressing, at the microscopic level, one or more of the components of a composite with a view towards the design and development of such bodies.

## 2) Table of Content

1) Forward	2
3) Statement of Problem Studied	4
4) Summary of the Most Important Results	5
4.1) Two-dimensional and three-dimensional analyses of pre-stressing composites	5
4.2) Interface/interphase studies: finite element analyses	14
4.3) Summary	20
5) Listing of Publication and Report	22
6) List of participating scientific personnel	23

### 3) Statement of Problem Studied

This proposal is aimed at understanding the effect of pre-stressing the constituents in the composites on enhancing overall load carrying capacity of the composites and examining the effect of microstructural morphology on the overall mechanical performance of composites. The study also incorporates more realistic constitutive models that include the effect of time-dependence in order to have a better understanding of the effect of pre-stress on the overall mechanical response of composites.

The project addressed the following problems:

- 1) Analyzed the overall performance of two composite systems comprising of a) brittle inclusions (glass, ceramics) in a ductile matrix (metals, polymers) and b) ductile inclusions in a brittle matrix. Linearized elastic response is considered for the brittle constituent while linearized viscoelastic constitutive model is used for the polymeric constituent. Both two-dimensional (2D) composites, having a circular inclusion, and three-dimensional (3D) composites with a spherical inclusion are studied. Exact analytical and numerical (finite element) solutions are presented.
- 2) Examined the effects of different microstructural arrangements and residual (pre-) stresses on the overall mechanical performance of the composites. The project also studied the discontinuities in field variables at the interfaces between particles and matrix, which is crucial in maintaining bonding between the constituents, and the effect of thermo- mechanical cycles during processing on the overall properties of the composites.

#### 4) Summary of the Most Important Results

The idea of prestressing, which is inducing internal stresses to improve the external load carrying capacity of structures, has been considered mainly with regard to large scale infrastructures such as bridges, aircraft fuselages, etc. Understanding the effect of prestressing the constituents on the overall mechanical performance of composite materials has rarely been carried out. This study starts with simplified microstructures of composites, i.e., a circular inclusion embedded in two-dimensional (2D) infinite matrix domain and a solid spherical inclusion placed in three-dimensional (3D) infinite matrix. Exact analytical solutions based on linearized elasticity and linearized viscoelasticity are obtained to understand the effects of prestressing and properties of the constituents on the overall performance of the composites. The main purpose of considering the simplified microstructures is to understand the feasibility in enhancing the load carrying capacity of composites by prestressing the constituents. The next step considers more realistic composite microstructures having several solid spherical particle arrangements dispersed in homogeneous matrix. Finite element (FE) method is used to obtain approximate solutions to the boundary value problems. The overall performance of composites with different interface/interphase behaviors and prestressed inclusions is examined.

##### 4.1) Two-dimensional and three-dimensional analyses of pre-stressing composites

The first composite system under consideration comprises of a brittle inclusion bonded to a ductile matrix. There are many examples of composite materials that belong to this category, such as glassy/carbon inclusion in polymer matrix, ceramic inclusion in metallic/polymeric matrix, brittle aggregate in asphalt matrix, etc. Consider a 2D composite, comprising of a solid circular cross-section, of a radius  $a$ , embedded in a matrix of infinite medium with a uniform constant traction  $T$  applied at the boundary. It is assumed that the inclusion behaves like linear elastic isotropic and homogeneous solids, while the matrix is comprised of a linear viscoelastic body. Both linear viscoelastic fluids and linear viscoelastic solids are considered for the matrix. It is further assumed that the inclusion and the matrix are perfectly bonded. The inclusion has a pre-existing compressive pressure  $P_o$ , termed as prestress. The solutions to field variables, i.e., deformation and stress fields in the inclusion and matrix, are determined by imposing equilibrium and compatibility condition and prescribing boundary condition. The normal stresses and displacements, which vary with the radial location,  $r$ , and time  $t$ , are summarized as:

$$\sigma_{rr}^i(r, t) = \sigma_{\theta\theta}^i(r, t) = P(t) - P_o \quad (1)$$

$$\sigma_{rr}^m(r, t) = T \left( 1 - \frac{a^2}{r^2} \right) + \frac{a^2}{r^2} P(t) \quad (2)$$

$$\sigma_{\theta\theta}^m(r, t) = T \left( 1 + \frac{a^2}{r^2} \right) - \frac{a^2}{r^2} P(t)$$

$$u_r^i(r, t) = \frac{r}{E^i} \left( \sigma_{\theta\theta}^i(r, t) - \nu^i \sigma_{rr}^i(r, t) \right) \quad (3)$$

$$u_r^m(r, t) = r \left[ J^m(t) \left( 2T - P(0) (1 + \pi^m) \right) - \int_0^t (1 + \pi^m) J^m(t-s) \frac{dP(s)}{ds} ds \right] \quad (4)$$

where the superscripts  $i$  and  $m$  denote the inclusion and matrix, respectively, the subscript  $rr$  or  $r$  and  $\theta\theta$  indicate the radial and circumferential (hoop) components, respectively. The elastic modulus and Poisson's ratio for the elastic inclusion are  $E^i$  and  $\nu^i$ , respectively, while  $J^m(t)$  and  $\pi^m$  are the time-dependent compliance and the contracting ratio for the viscoelastic matrix. The contracting ratio corresponds to Poisson's effect in linear elastic materials. The radial stress at the interface is  $P(t)$ , which varies with time due to the viscoelastic response of the matrix. For the problem under consideration, the shear components of stresses and strains are absent.

The time-dependent function for the matrix compliance depends on the viscoelastic behaviors of the materials. In this study, the linear Maxwell fluid model is chosen for the viscoelastic fluid-like matrix, while the Standard Linear Solid (SLS) model is used for the viscoelastic solid-like matrix. The time-dependent compliance and radial stress at the interface when the Maxwell fluid is considered for the matrix are:

$$J^m(t) = \frac{1}{E} + \frac{t}{\mu} \quad (5)$$

$$P(t) = \frac{E(1-\nu^i) \left( P_o(1-\pi^m) - 2T \right)}{\left( E(1-\nu^i) + E^i(1-\pi^m) \right) (1+\pi^m)} \exp \left[ - \frac{E^i E (1+\pi^m)}{\mu \left( E(1-\nu^i) + E^i(1+\pi^m) \right)} t \right] + \frac{2}{1+\pi^m} T \quad (6)$$

The time-dependent compliance and radial stress at the interface when the SLS model is considered for the matrix are:

$$J^m(t) = \frac{1}{E} - \frac{E_o}{E(E+E_o)} \exp \left[ - \frac{EE_o}{\mu(E+E_o)} t \right] \quad (7)$$

$$P(t) = \frac{E_o E^i (1-\nu^i) \left( P_o(1+\pi^m) - 2T \right)}{\left( E(1-\nu^i) + E^i(1+\pi^m) \right) \left( (E+E_o)(1-\nu^i) + E^i(1+\pi^m) \right)} \exp \left[ - \frac{E_o \left( E(1-\nu^i) + E^i(1+\pi^m) \right)}{\mu \left( (E+E_o)(1-\nu^i) + E^i(1+\pi^m) \right)} t \right] - \frac{2E^i(1-\pi^m)}{E(1-\nu^i) + E^i(1+\pi^m)} T + \frac{E(1-\nu^i)}{E(1-\nu^i) + E^i(1+\pi^m)} P_o \quad (8)$$

The stress and displacement fields are finally determined by substituting the stress  $P(t)$  from either Eq. (6) or Eq. (8) into Eqs. (1)-(4). In Eqs. (5)-(8),  $E$ ,  $E_o$ , and  $\mu$  are the material parameters for the viscoelastic matrix.

In a manner similar to the discussion above, the response of the composite in 3D domain can be obtained. Consider a 3D composite, comprising of a solid sphere, of a radius  $a$ , embedded in a matrix of infinite medium with a uniform constant traction  $T$  applied at the boundary. Like in the 2D problem, it is assumed that the inclusion behaves like linear elastic isotropic and homogeneous solids, while the matrix is comprised of a linear viscoelastic body and that the inclusion and the matrix are perfectly bonded. The same variables and material properties used in the 2D case are considered for the 3D composites. A detailed discussion of the stresses and displacements response for 3D composite problems is given in Zhu et al. (2013a). It is noted that the real composite systems occupy a 3D space, thus a 3D solution is more realistic than 2D solution. However, 2D solutions are generally simpler and have been widely used in obtaining rough estimates of the response of composites prior to designing the composite materials and/or when designing structures made of composites. One of the intentions of this study is to examine the responses from both 2D and 3D cases which provide justification of the use of simpler 2D analyses.

The following cases are preliminary studies on the effect of prestressing the inclusion on the overall performance of the composites discussed above.

**Case 1:** Responses of composites with the Maxwell (MX) and SLS models for the matrix.

Consider a composite with a uniform external tension  $T=200$  MPa prescribed on the boundary of the infinite matrix and a compressive prestress  $P_o=200$  MPa prescribed on the inclusion. Table 1 presents the material parameters for the inclusion and viscoelastic matrix, both with Maxwell and SLS models.

Table 1 Properties of the constituents

Constituents	Parameters	Values
Elastic inclusion	$E^i$	200 GPa
	$\nu^i$	0.25
Maxwell matrix	$E$	100 GPa
	$\pi^m$	0.33
	$\mu$	69 GPa.s
SLS matrix	$E$	50 GPa
	$E_o$	50 GPa
	$\pi^m$	0.33
	$\mu$	69 GPa.s

Figures 1 and 2 depict the hoop and radial stress fields, respectively, in the composites at several instants of time: early time, some time during relaxation, and later time, obtained from 2D and 3D analyses. Responses from the Maxwell and SLS viscoelastic matrix are also shown. It is seen that the differences in the hoop stresses from the 2D and 3D analyses are quite significant, especially for the hoop stress discontinuity at the interface, while the radial stresses from the 2D

and 3D analyses are sufficiently close. High values in the hoop stress discontinuities can lead to debonding between the inclusion and matrix, reducing the load carrying capacity of the composites. Although 2D analyses are typically less complicated, they might lead to inaccurate prediction of the problems. Especially in this case, if one wants to understand the debonding issues at the interface and enhance the load carrying capacity by avoiding debonding, it is perhaps necessary to consider 3D analyses. Next, it is also observed that for the above composites under external tensile load, the magnitude of the tensile stress in the brittle inclusion and in the matrix decreases significantly by introducing the compressive prestress, as illustrated in Figs. 3 and 4. Reducing the tensile stresses in the brittle inclusion and ductile matrix can delay or minimize failure in the constituents due to cracking. It is also seen that when the Maxwell fluid model is considered, the matrix undergoes significant relaxation than when the SLS model is considered, which is expected since the Maxwell fluid model experiences continuous relaxation.

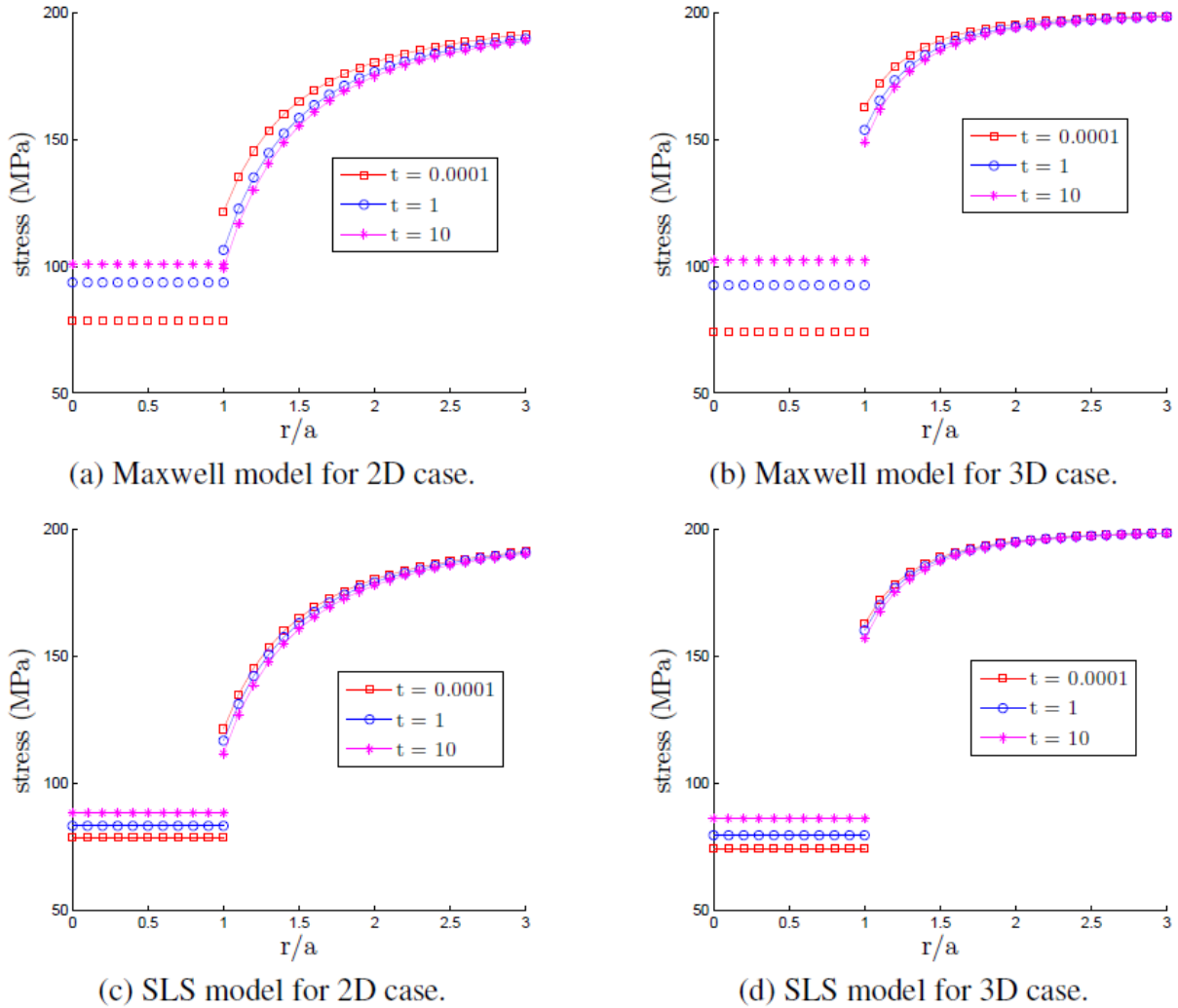
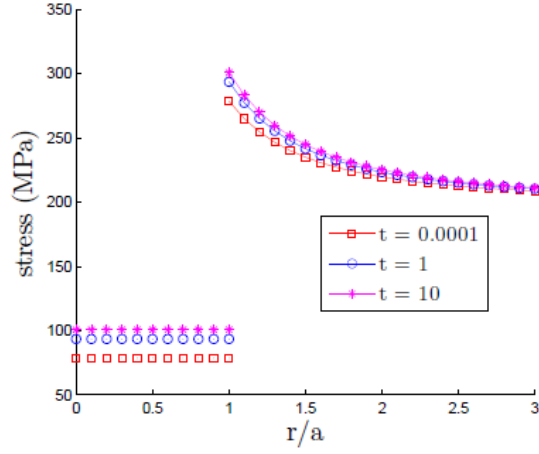
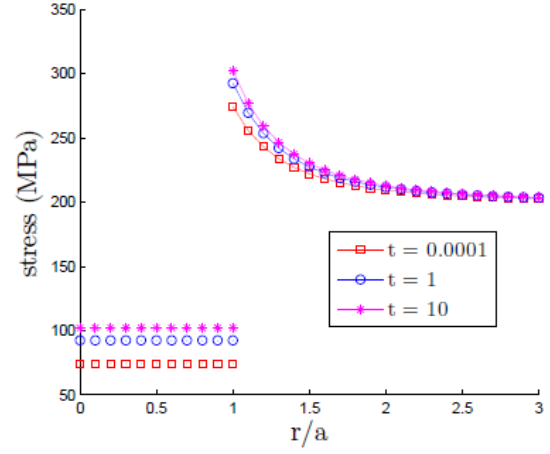


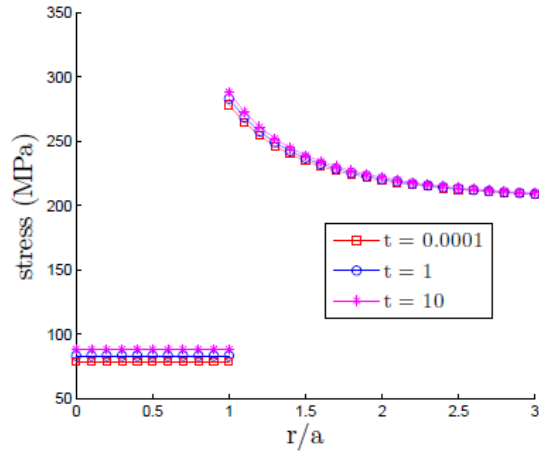
Figure 1 Hoop stresses along the radial direction at several instants of time.



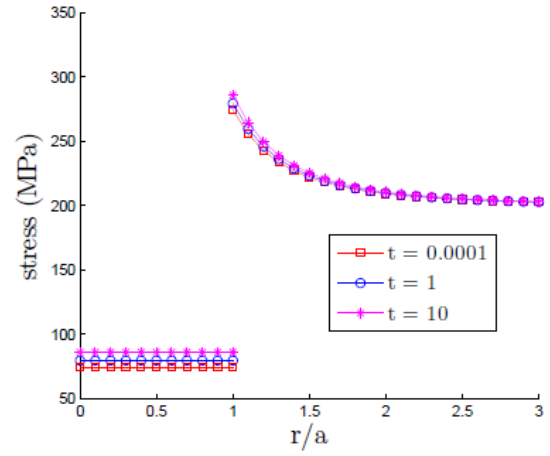
(a) Maxwell model for 2D case.



(b) Maxwell model for 3D case.

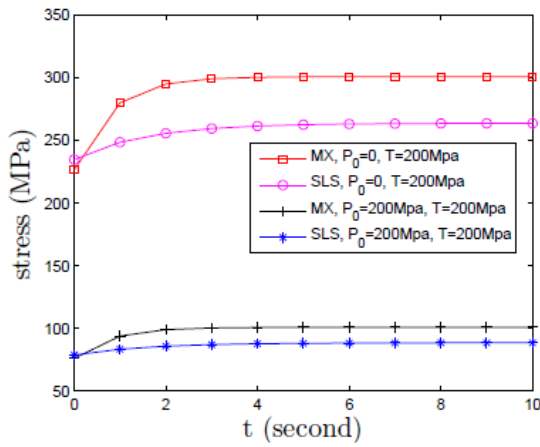


(c) SLS model for 2D case.

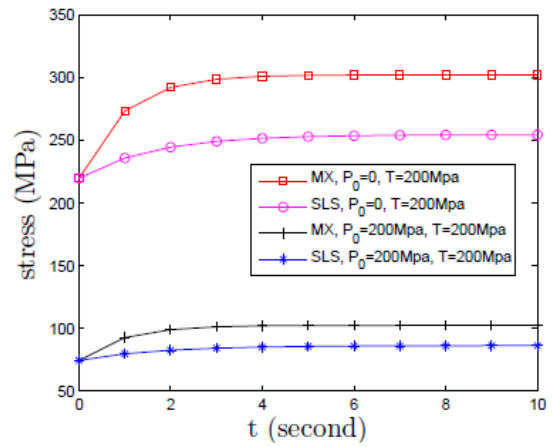


(d) SLS model for 3D case.

Figure 2 Radial stresses along the radial direction at several instants of time



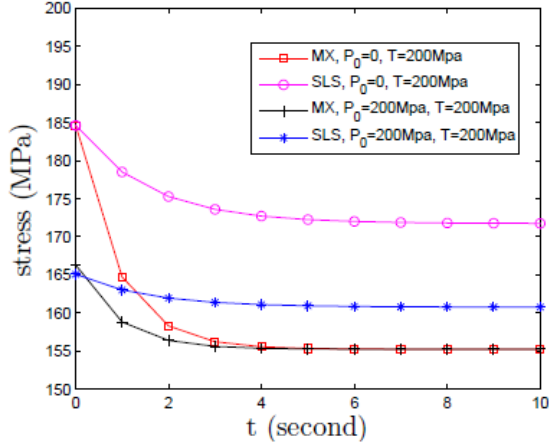
(a) Stresses in the inclusion for 2D case.



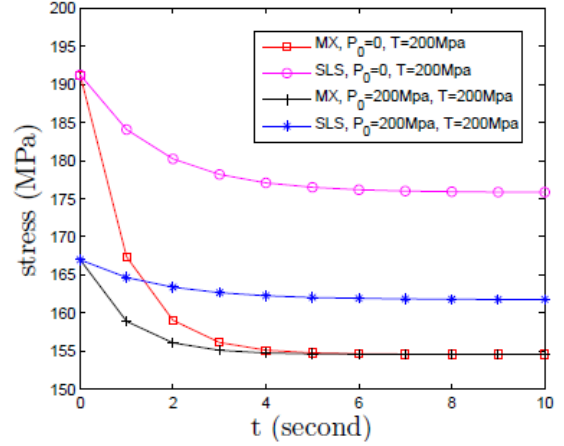
(b) Stresses in the inclusion for 3D case.

Figure 3 Hoop stress in the inclusion with and without pre-stressing the inclusion.





(a) at  $r = 1.5a$  for 2D case.



(b) at  $R = 1.5a$  for 3D case.

Figure 4 Hoop stress in the matrix with and without prestressing the inclusion.

**Case 2:** The effects of prestresses and instantaneous (elastic) modulus of the matrix

As both Maxwell and SLS show similar trends in terms of the stress fields in the composites, in this section the discussion is confined to the Maxwell viscoelastic matrix. From Eq. (1) it is seen that increasing the value of the compressive prestress  $P_0$  would lead to high compressive stresses in the brittle inclusions, which is desired if the composites are predominantly carrying the external tensile stresses. This can avoid cracking in the brittle inclusions. However, the load carrying capacity in the composite can reduce due to debonding/delamination between the inclusion and matrix. As stated above, debonding occurs due to high stress discontinuities at the interface between the matrix and inclusion,  $\Delta\sigma_{\theta\theta}(a, t) = \sigma_{\theta\theta}^m(a, t) - \sigma_{\theta\theta}^i(a, t)$ . Figures 5 shows the hoop stress discontinuities at the interface, at different times, when different values of prestress are considered. In this example, there is a range of pre-stress values at which the stress discontinuity is minimum, e.g. 50-200 MPa. The result from this case study can help designing composite with improved load carrying capacity.

Furthermore, the stiffness of the constituents also influences the magnitude of stresses in the composites. Figure 6 shows the normal stress field in the inclusion and the hoop stress discontinuities at the interface when the viscoelastic matrix has different values of instantaneous modulus. As expected the instantaneous modulus of the viscoelastic matrix only affects the response at early time. The softer the matrix, the smaller the stress discontinuity at the interface is. Thus, when a brittle and stiffer inclusion is used for composites that are designed to sustain external tensile stresses, applying compressive prestress in the inclusion and considering softer or compliant matrix hold promise with regard to enhancing load carrying capacity of composites.

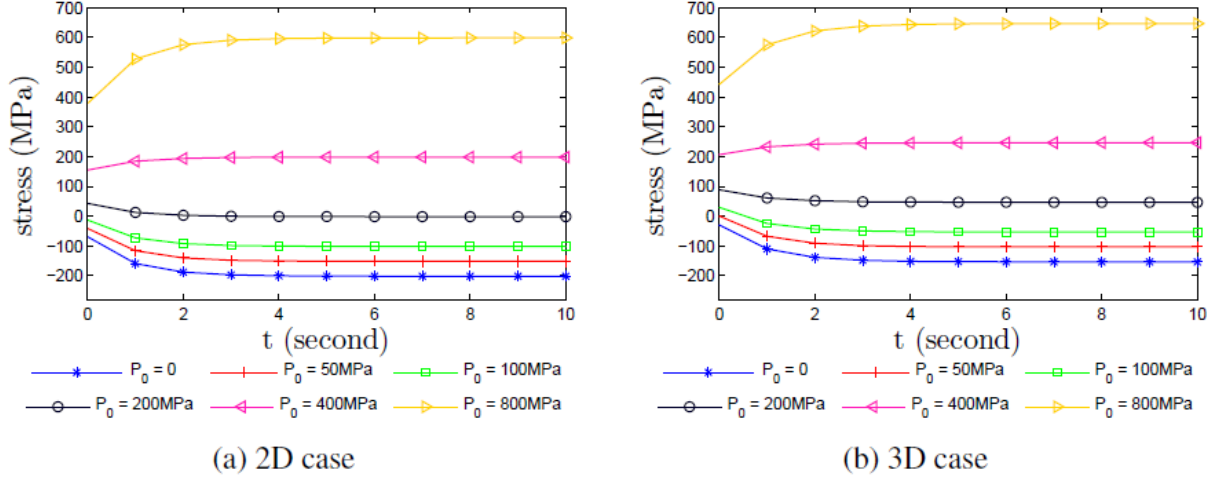


Figure 5 Hoop stress discontinuities at the interface under different prestress values.

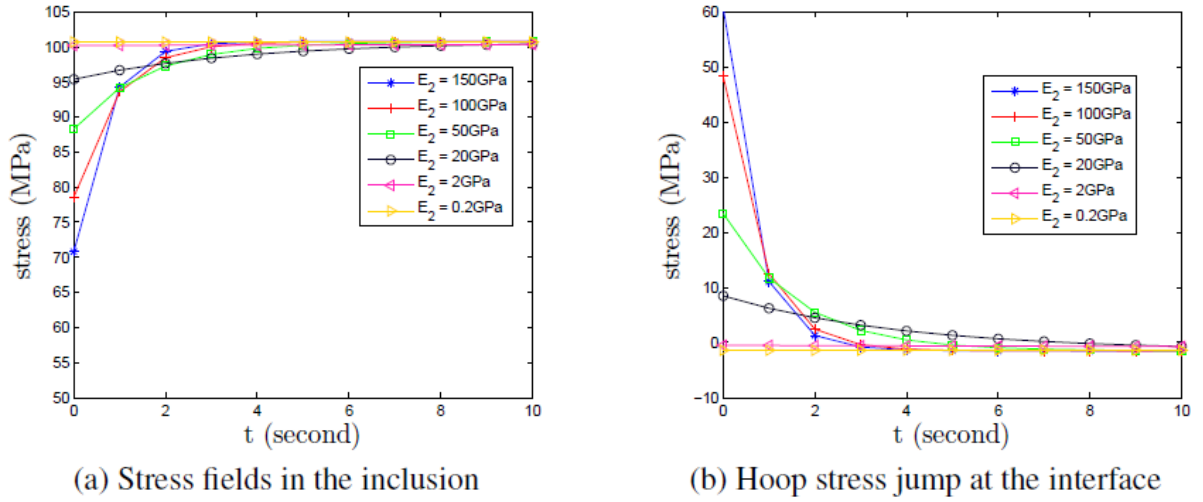


Figure 6 The effect of elastic (instantaneous) modulus of the matrix on the stress field.

### Case 3: Response of the composite due to uniaxial tensile load

The above case studies discuss the response of composites when a uniform tensile stress is prescribed on the entire boundary of the composites. In many applications, composites are often subjected to non-uniform boundary conditions, such as uniaxial loadings. This case study considers the effect of prestressing the inclusion on the overall response of composites when the composite system is subjected to a uniaxial tensile load. In such situation, the stresses acting on the inclusion and matrix depend not only on the radial direction, but also vary in the circumferential (hoop) direction. Furthermore, the composite experiences both normal and shear stresses. Figure 7 illustrates the maximum principal stresses in the inclusion at two radial locations,  $r=0.5a$  and  $r=a$ , at time  $t=10s$ , and at various circumferential direction, measured by an angle  $\theta$ . The angle  $\theta$  is measured starting from the horizontal axis pointed to the right direction and going counter clock wise direction up to  $\theta=2\pi$  rad=6.28 rad. It is seen that the stresses in the inclusion vary from tension to compression. Figure 8 shows the maximum shear stresses at the inclusion,  $r=0.5a$ , and at the matrix,  $r=1.5a$ . It is seen that the existence of the shear stresses in

the constituents could lead to shear failure in the composites. In order to examine the potential of debonding at the interface, the hoop stress discontinuities are also examined, as shown in Fig. 9. Unlike the case of uniform external tensile stress, when the composite is subjected to a uniaxial tensile load the hoop stress discontinuities at the interface between the inclusion and matrix do not necessarily drop with increasing the compressive prestress. At certain angles, e.g.  $\theta=0$ , the stress discontinuity is small, whereas at other angles, e.g.  $\theta=\pi/2$ , the stress discontinuity increases. Although in the case of uniaxial loading, the stress discontinuities would vary at different locations and prestressing the inclusion does not necessarily reduce the entire stress discontinuities, certain choice of prestressing can still give an enhanced performance when taking into account the overall response of the composites. It is perhaps necessary to consider non-uniform prestress in the case of uniaxial loading, as discussed in Zhu et al. (2013a).

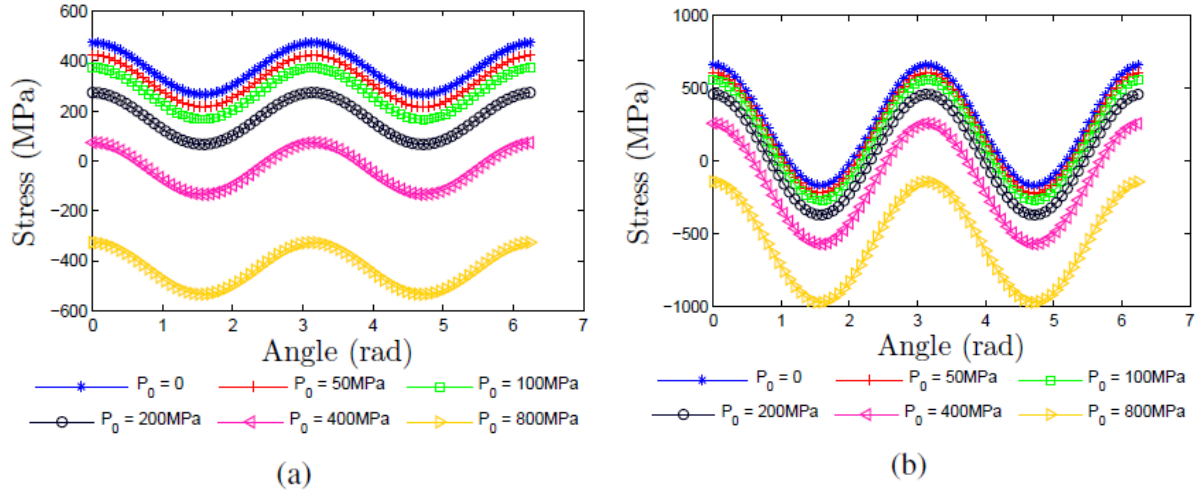


Figure 7 The maximum principal stresses in the inclusion at a)  $r=0.5a$  and b)  $r=a$

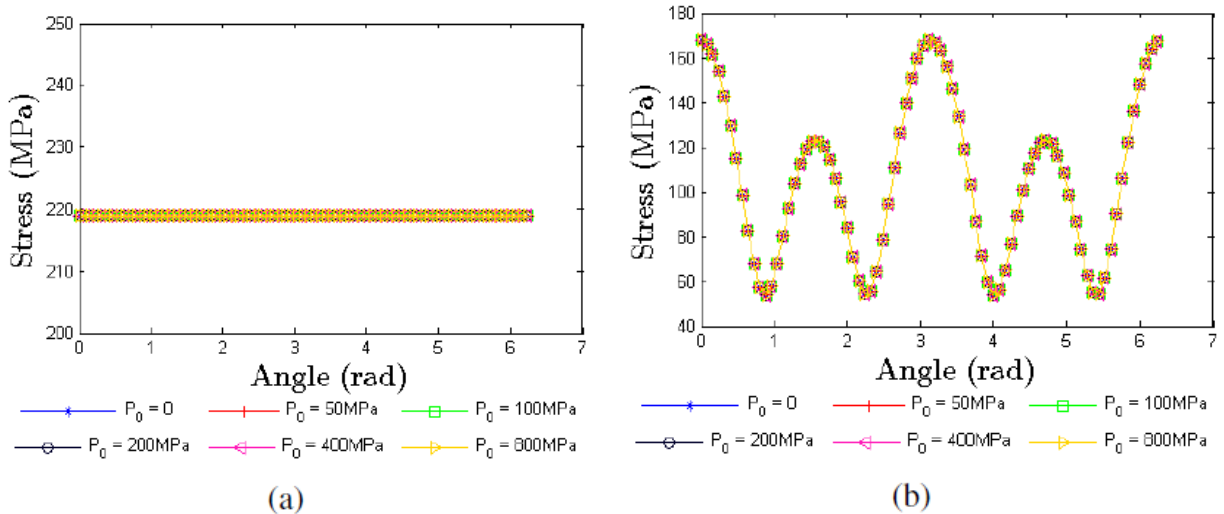


Figure 8 The maximum shear stresses a) in the inclusion  $r=0.5a$  and b) in the matrix  $r=1.5a$

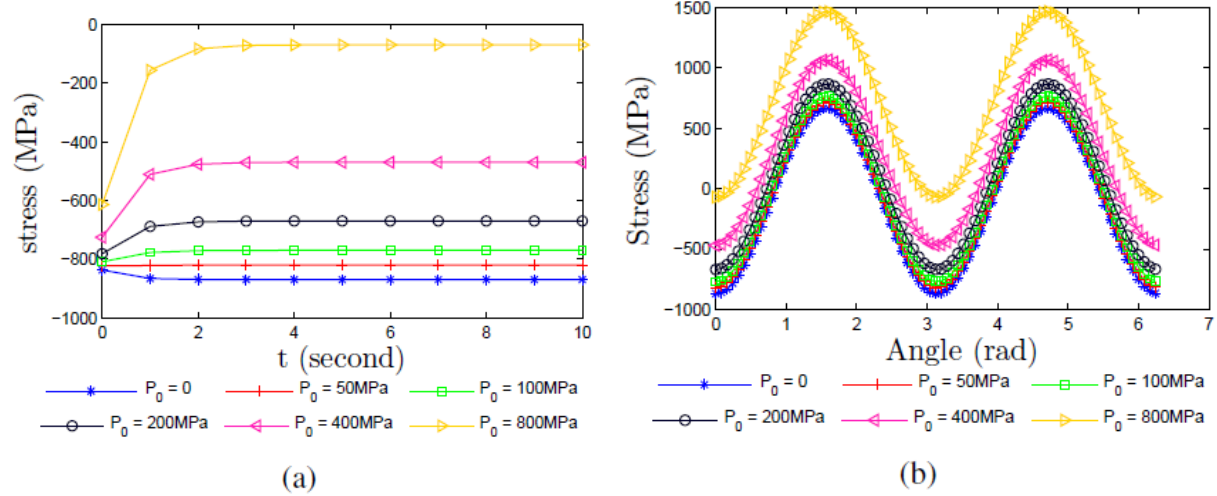


Figure 9 Hoop stress discontinuities at the interface a) at  $\theta=0$  and b) at  $t=10s$

#### Case 4: Prestressing ductile inclusion in a brittle matrix

There are other types of composites which comprise of a ductile inclusion embedded in a brittle matrix, such as adding rubber particles to the glassy polymers or using ductile inclusions in the brittle ceramic matrix in order to improve ductility of the composites. This case study deals with a composite with a ductile inclusion placed in an infinite medium of a brittle matrix, and the composite is subjected to a uniform compressive stress,  $T=-200$  MPa, on its boundary. Like in the previous cases, both inclusion and matrix are assumed isotropic, homogeneous, and undergo small deformation, and the interface between the inclusion and matrix is perfectly bonded. Only 3D solution is considered here. The inclusion is modeled as a Maxwell viscoelastic fluid while the matrix is assumed to be linearized elastic solid. The properties for the Maxwell viscoelastic fluid and elastic solid from Table 1 are used. Figure 10a depicts the time-dependent radial (and hoop) stresses in the inclusion at various values of prestress. Both tensile and compressive prestresses are considered. It is seen that significant compressive stress relaxation occurs in the viscoelastic inclusion, which should be expected. The hoop stresses in the brittle matrix at the location  $r=1.5a$  is shown in Fig. 10b. It is shown that prestressing the inclusion in this case does not have significant effect on the stress response of the viscoelastic inclusion as well as the elastic matrix. The hoop stress discontinuities at the interface, shown in Fig. 11, indicate that the stress discontinuities reach a minimum value when the prestress is in the range of 400-800 MPa.

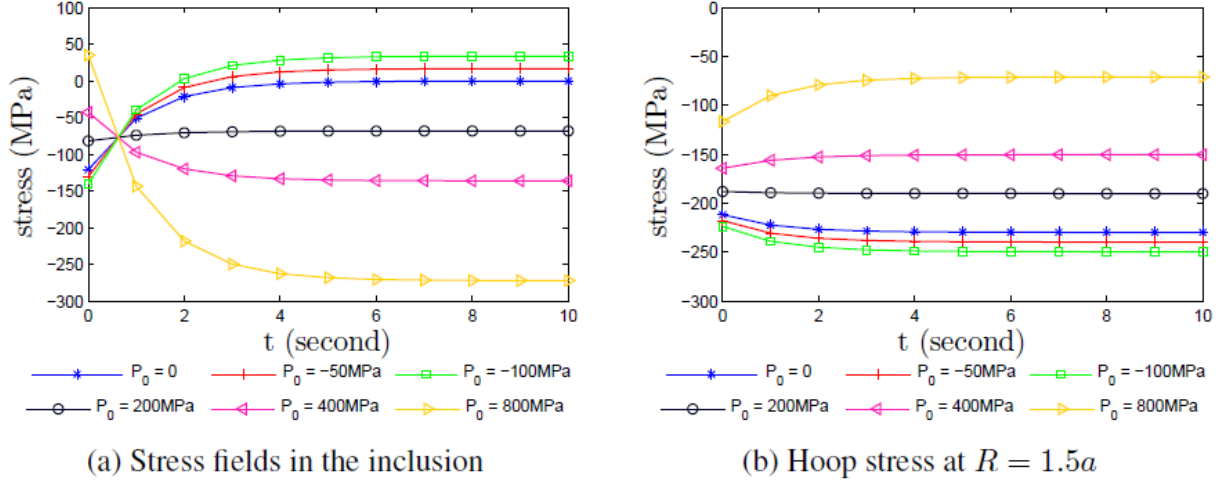


Figure 10 Stress fields in the inclusion and matrix at various values of prestresses.

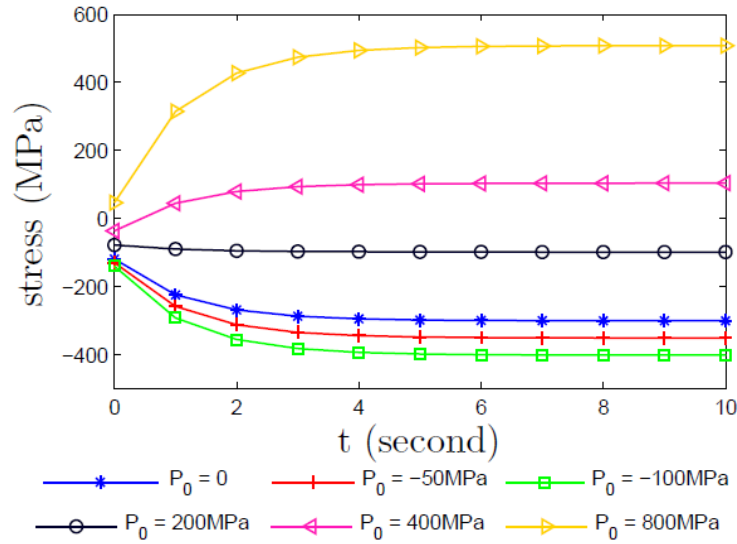


Figure 11 Hoop stress discontinuities at the interface at different values of prestress

#### 4.2) Studies of interface/interstitial matter: finite element analyses

In reality composite microstructures comprise of several particle arrangements dispersed in matrix medium and the microstructures might also contain other inhomogeneities, such as voids, cracks, and other constituents due to chemical reactions during the processing. When one wants to model and analyze response of composites, several simplification and approximation are often made in order to reduce complexity as it might not be possible or even necessary to include all inhomogeneities in characterizing the overall response of the composites. In most cases, it might not be possible to obtain exact closed form solutions in predicting the response of the composites when certain complexity is considered. For this purpose, approximate (numerical) solutions are often sought. Finite element (FE) method is commonly used to obtain approximate solutions to predict the overall response of composites when a more complex microstructure is considered.

This section discusses FE analyses of composites due to a prestress effect by incorporating more realistic microstructures, but still within certain simplification and approximation. Consider a 3D cubic domain of a composite with a side length of 0.01mm. The composite contains uniform solid spherical inclusions, each of radius 0.001mm, dispersed randomly in the matrix domain. Figure 12 illustrates the composite microstructures, having particle and matrix constituents, with 10 and 30% particle volume contents. The interfaces between particles and matrix are assumed perfect. Composites could also contain a small region (interstitial matter) between the particle and matrix, which is considered as a third constituent. Interstitial matter could be formed from the reaction process between the inclusions and matrix during the manufacturing of the composites, or surface treatment or coating could also be performed on the inclusions in order to enhance the bond strength between the inclusions and matrix. This study also considers composite microstructures with interstitial regions between the particles and matrix, as shown in Fig. 13. The thickness of the interphases is assumed uniform, which is 0.0002 mm.

FE analyses are conducted on composites, having alumina (brittle) particles of a linear elastic material model dispersed in FM73 polymeric matrix of viscoelastic solid (SLS) model, subjected to a uniform tensile load on the boundaries of the composites. The properties of the alumina, interstitial matter, and FM73 polymer are listed in Tables 2 and 3. The interstitial matters are assumed as linear elastic. It is noted that experimentally characterizing the properties and response of the interstitial matter is rarely done, and the interstitial matter would strongly depend on the processing method, and the inclusions and matrix used in the composites. Since the properties of interstitial matters are not readily available in the literature, parametric studies are conducted on examining the effects of interstitial matters of various properties and prestress on the overall response of composites.

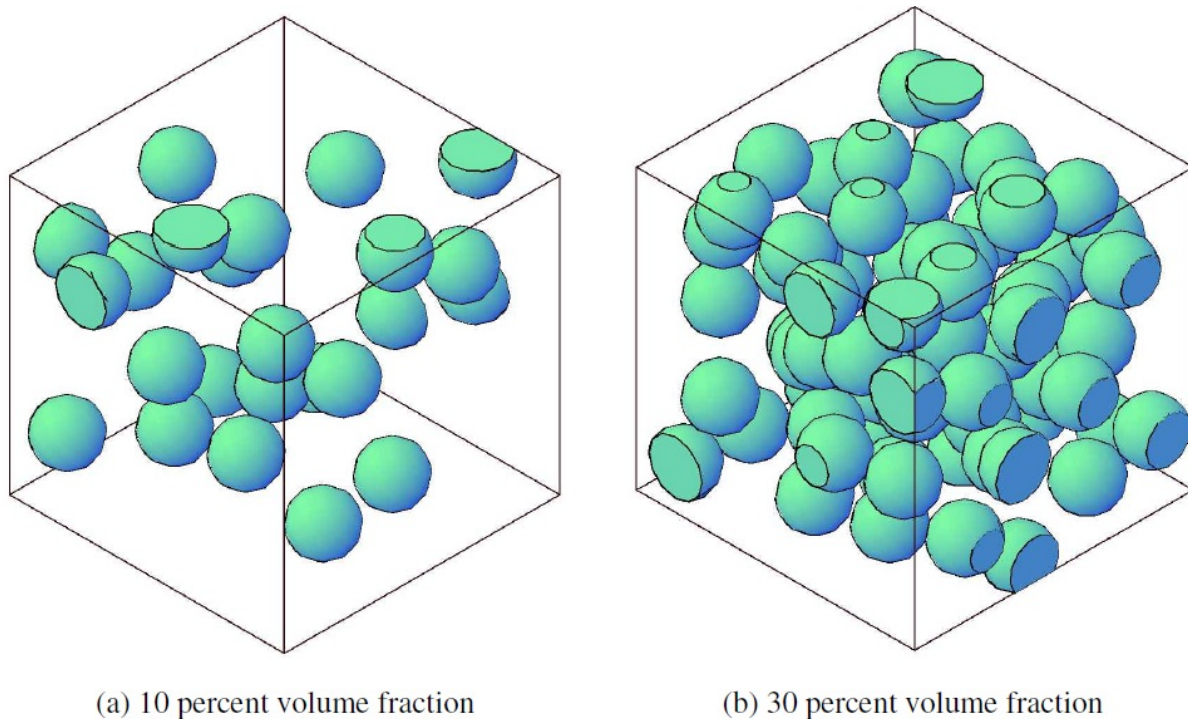


Figure 12 3D composite models with solid spherical particles



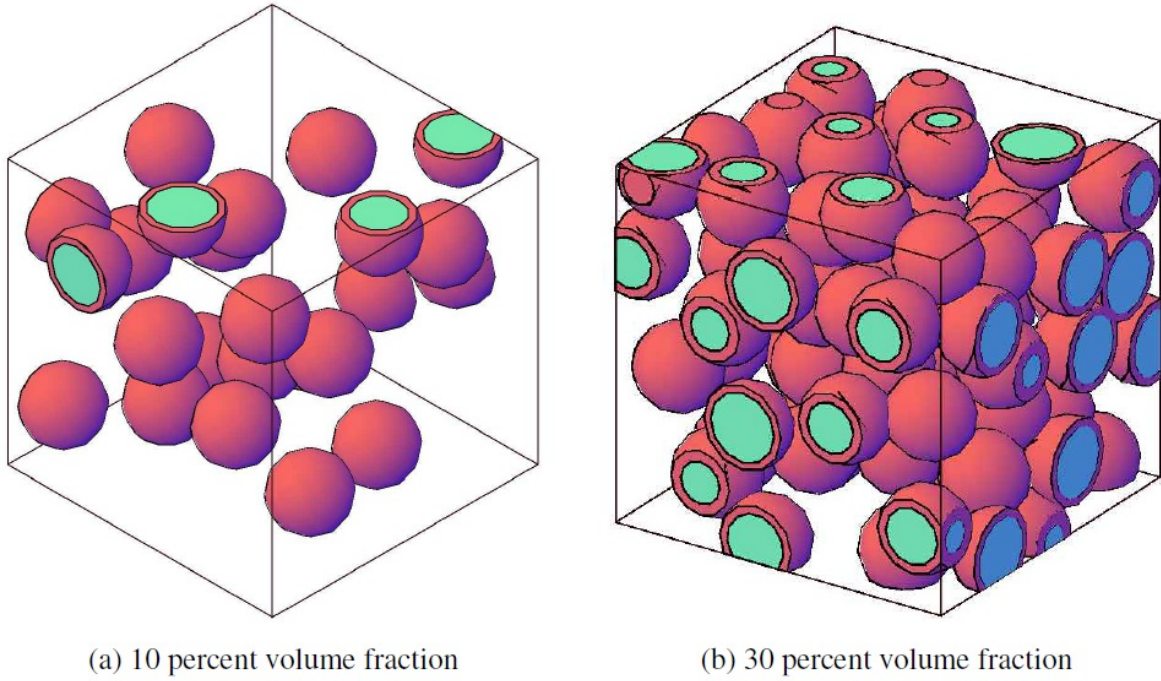


Figure 13 3D composite models with solid spherical particles and interstitial regions

Table 2 Linear thermo- elastic properties of constituents

Constituents	$E$ (GPa)	$\nu$	$\alpha(^{\circ}\text{C})$
Inclusions	200	0.25	$4.6 \times 10^{-6}$
Matrix	2.71	0.35	$6.5 \times 10^{-5}$
Stiff interstitial matter	400	0.25	-
Soft interstitial matter	1.0	0.25	$2.0 \times 10^{-4}$

Table 3 Time-dependent properties of polymeric matrix

$n$	$\lambda_n(s^{-1})$	$D_n \times 10^{-6}(MPa)^{-1}$
1	1	21.0
2	$10^{-1}$	21.6
3	$10^{-2}$	11.8
4	$10^{-3}$	15.9
5	$10^{-4}$	21.6
6	$10^{-5}$	20.0
$E = 2710 MPa \quad \nu = 0.35$		

### Case 5: The effect of particle volume contents and properties of the interstitial matters

This study examines the stress distributions in the inclusion and matrix, with and without prestressing the inclusions. Consider composites with 10 and 30% particle volume contents, subjected to a tri-axial tensile stress of 60 MPa. Figure 14 illustrates the maximum principal stress distributions in the particles, at time=10s, when the particles are under a uniform pre-stress of 200MPa. The responses are also compared to the ones without pre-stress. As expected, prestressing the brittle inclusions would induce compressive stresses in the inclusions, while its effect on the stress distributions in the matrix (Fig. 14) is negligible. It is also seen that in the composite with higher particle contents, there are some localized stresses both in the particles and matrix, which could be due to relatively small distances between particles in the matrix.

Next, the effect of properties of the interstitial matters on the overall response of composite is studied. Both stiff and soft interphases are considered. The properties of the interphases are given in Table 2. The interfaces between the particles, interphases, and matrix are assumed perfectly bonded. The prestress is done on the particles. Figure 15 illustrates the stress distributions in the particles, interphases, and matrix. It is seen that the role of the interphases could be significant in altering the overall performance of the composites. When the interphases are stiffer than the particles the matrix experiences higher stresses as compared to the composites with soft interphases. The interphases also influence the amount of the compressive stresses in the brittle particles. The corresponding maximum strain contours for the composites with stiff and soft interphases, at early and later times, are given in Fig. 16. As expected there strains increase with times due to the viscoelastic matrix. It is noted the stress contours remain nearly the same as a result of imposing equilibrium conditions. Continuously increasing strains with time could eventually also lead to delamination between inclusions and matrix.

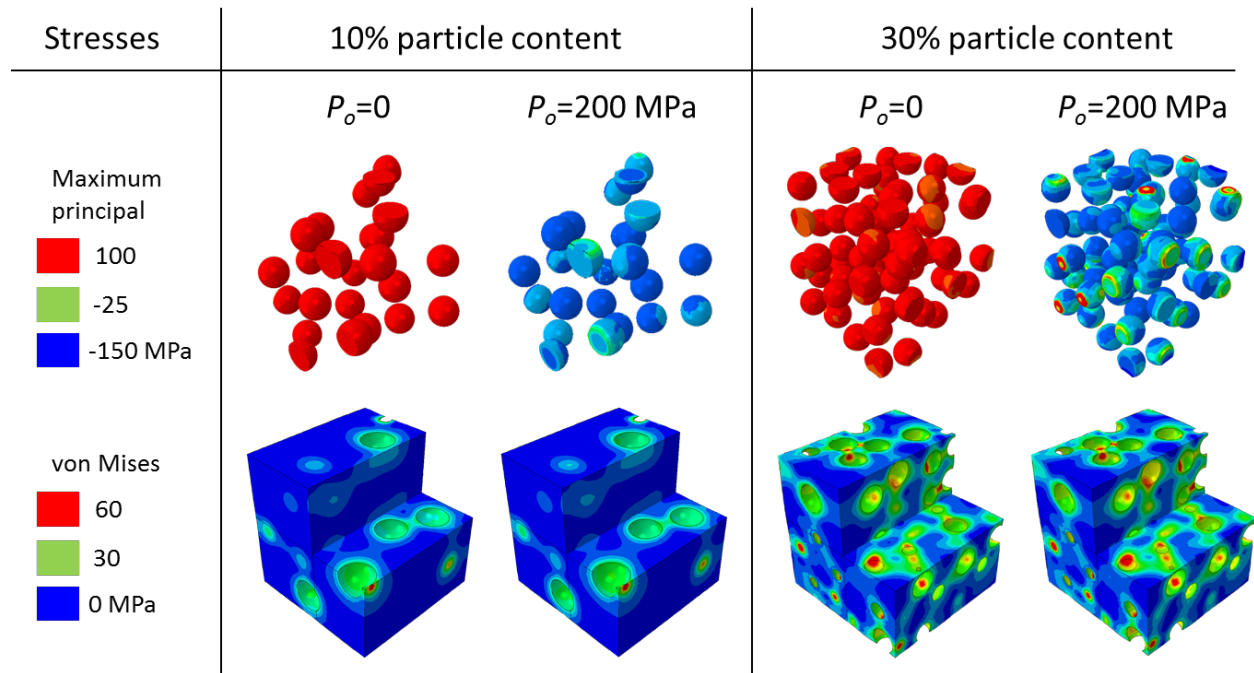


Figure 14 The effect of particle volume contents on the overall response of composites ( $t=10s$ )



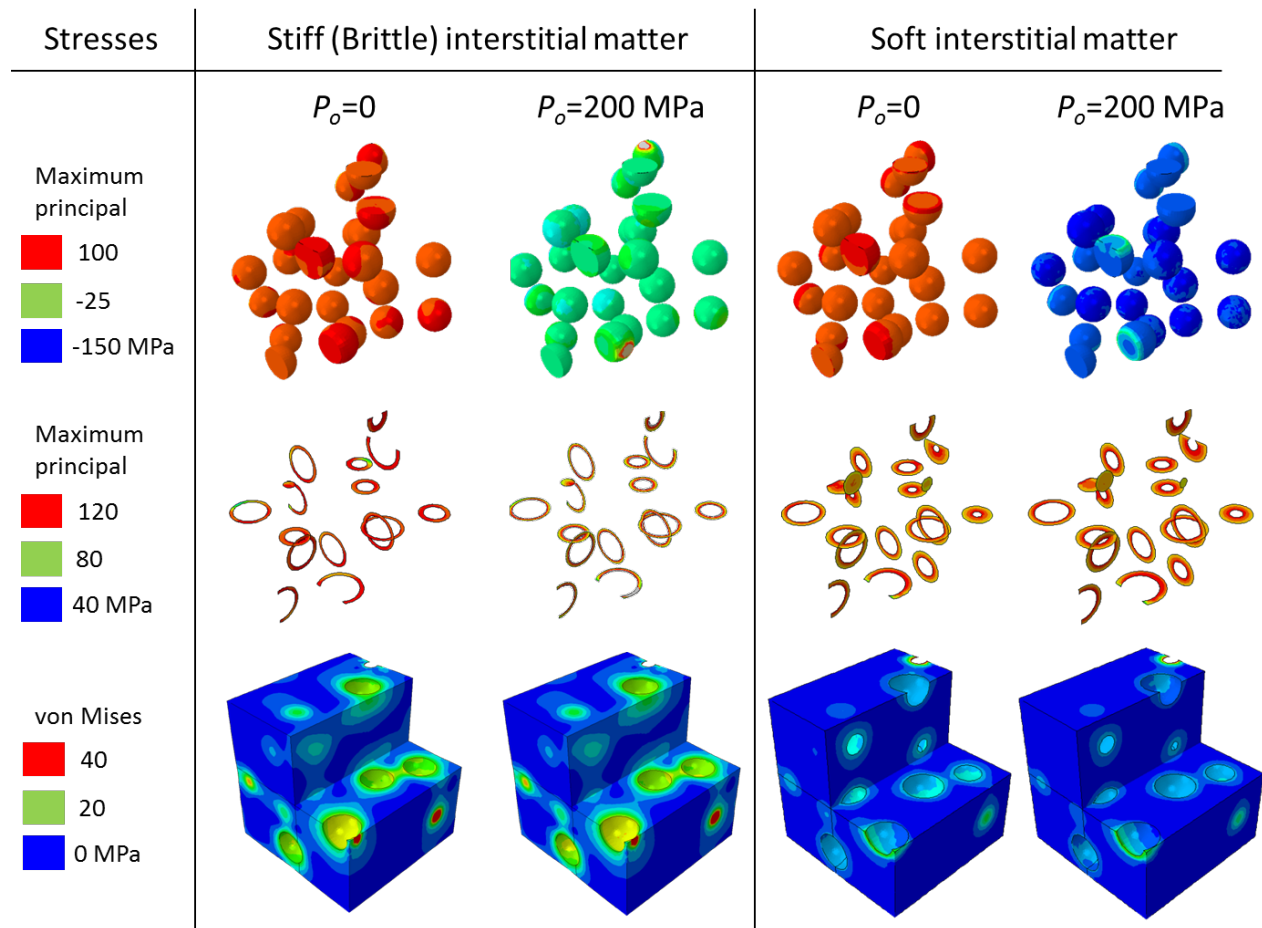


Figure 15 The effect of stiff and soft interphases on the overall response of composites with 10% particle volume content ( $t=10s$ ). Top: inclusions; middle: interstitial matters; bottom: matrix

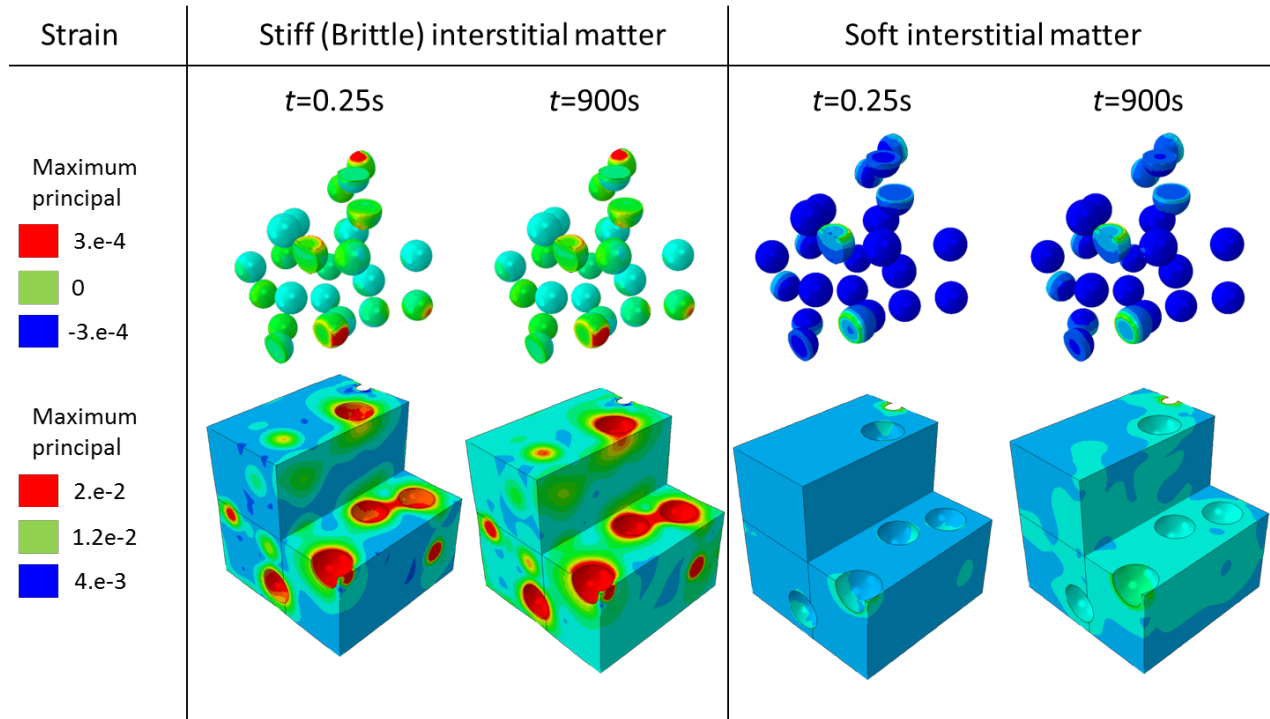


Figure 16 Maximum principal strain contours in composites with 10% particle volume content at early and later times. Top: inclusions; bottom: matrix.

#### Case 6: Prestressing particles through thermo-mechanical process

All of the above cases considered the effect of prestress and constituent behaviors on the overall performance of composites. This case shows a possible method to induce prestress on the inclusions by temperature changes. Consider a composite comprising of brittle inclusions coated by soft polymers, called soft interphases, dispersed in polymeric matrix. The linear thermo-elastic properties of the constituents are given in Table 2. When a processing method involves curing at an elevated temperature, e.g., 225°C, upon cooling down to room temperature, the mismatches in the thermal expansion coefficients ( $\alpha$ ) in the inclusions, interphases, and matrix induce compressive prestresses to the inclusions. Figure 17 shows the contours of the maximum principal stresses in the constituents at room temperature, after the cooling process. It is seen that all inclusions are under compressive stresses, the soft interphases experience tension, while the matrix is under nearly zero stresses. It is also observed that the shear stresses in the constituents are negligible. This case study shows a possibility in prestressing the inclusions through temperature changes.

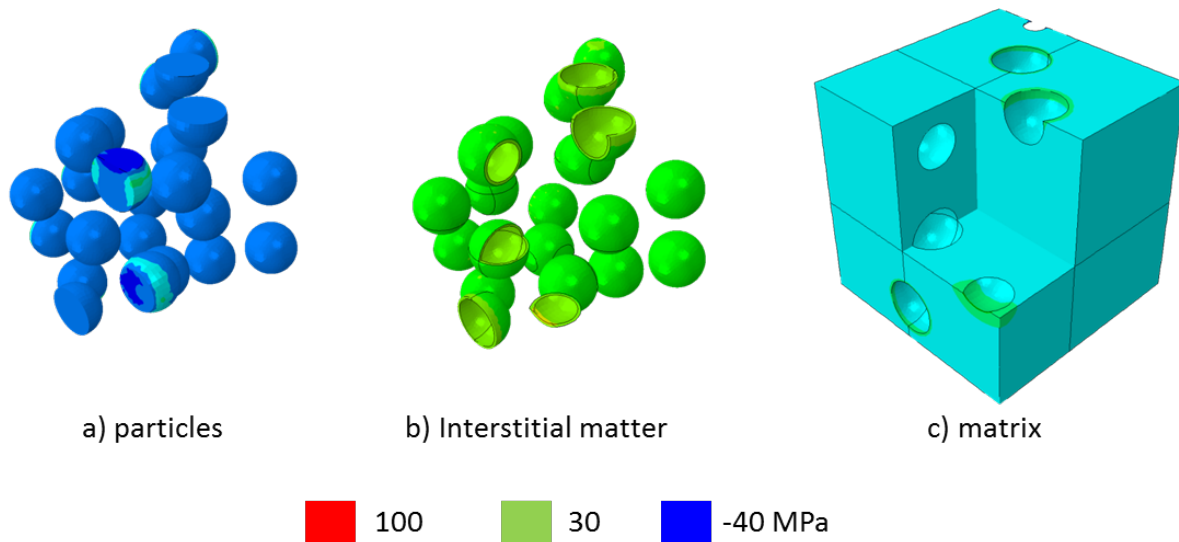


Figure 17 Maximum principal stresses in the constituents due to a temperature change.

#### 4.3) Summary

When composites comprising of brittle inclusions and ductile/soft matrix are considered, high values in the hoop stress discontinuities are observed. These high stress discontinuities can lead to debonding between the inclusion and matrix, reducing the load carrying capacity of the composites. It is also observed that for the above composites under uniform external tensile load, the magnitude of the tensile stress in the brittle inclusion and in the matrix decreases significantly by introducing the compressive prestress to the brittle inclusions. Reducing the tensile stresses in the brittle inclusion and ductile matrix can delay or minimize failure in the constituents due to cracking. Furthermore, the stiffness of the constituents also influences the magnitude of stresses in the composites. As expected the instantaneous modulus of the viscoelastic matrix only affects the response at early time. The softer the matrix, the smaller the stress discontinuity at the interface is. Thus, when a brittle and stiffer inclusion is used for composites that are designed to sustain external uniform tensile stresses, applying compressive prestress in the inclusion and considering softer or compliant matrix hold promise with regard to enhancing load carrying capacity of composites. Unlike the case of uniform external tensile stress, when the composite is subjected to a uniaxial tensile load the hoop stress discontinuities at the interface between the inclusion and matrix do not necessarily drop with increasing the compressive prestress. At certain locations, the stress discontinuity is small, whereas at other locations the stress discontinuity increases. Although in the case of uniaxial loading, the stress discontinuities would vary at different locations and prestressing the inclusion does not necessarily reduce the entire stress discontinuities, certain choice of non-uniform prestressing can still give an enhanced performance when taking into account the overall response of the composites.

FE analyses of composites due to prestress are presented by incorporating more realistic microstructures, but still within certain simplification and approximation. The composite contains uniform solid spherical inclusions of a brittle material, each of radius 0.001mm,

dispersed randomly in the viscoelastic matrix domain. Prestressing the brittle inclusions induces compressive stresses to the brittle inclusions, while its effect on the stress distributions in the matrix is negligible. Thus, prestressing the inclusions is promising in increasing tensile load carrying capacity in composites comprising of brittle inclusions. In the composite with higher particle contents, there are some localized stresses both in the particles and matrix, which could be due to relatively small distances between particles in the matrix. High localized stresses could induce failure in the composites, reducing the load carrying capacity of the composites. Next, the effect of the properties of interstitial matters on the overall response of composite is studied. The role of the interstitial matters is significant in affecting the overall performance of the composites. When the interstitial matters are stiffer than the particles the matrix experiences higher stresses as compared to the composites with soft interphases. The interstitial matters also influence the amount of the compressive stresses in the brittle particles. Moreover, strains increase with time due to the viscoelastic matrix and continuously increasing strains with time could eventually also lead to delamination between inclusions and matrix.

Finally, the FE analysis was conducted so as to examine a possible method to induce prestress on the brittle inclusions, coated by soft polymer interphases, by temperature changes. When a processing method involves curing at an elevated temperature, upon cooling down to room temperature, the mismatches in the thermal and mechanical properties of the inclusions, interphases, and matrix induce compressive prestresses in the inclusions, while the soft interphases experience tension and the matrix is under nearly zero stresses. It is also observed that the shear stresses in the constituents are negligible. This study shows that prestressing brittle inclusions could lead to increasing load carrying capacity of composites under tensile loading.

## 5) Listing of Publication and Report

### Journal articles:

Zhu H, Muliana A, and KR Rajagopal, “Effect of Prestress on the Mechanical Performance of Composites” under review

Zhu H, Muliana A, and KR Rajagopal, “FE analysis of effect of prestress and interphases on the performance of composites,” in preparation

## 6) List of participating scientific personnel

Principal Investigators: Anastasia Muliana and KR Rajagopal

Graduate Student: Huanlin Zhu